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# The Problem of Mass\*

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## Abstract

The quark-lepton mass problem and the ideas of mass protection are reviewed. The Multiple Point Principle is introduced and used within the Standard Model to predict the top quark and Higgs particle masses. We discuss the lightest family mass generation model, in which all the quark mixing angles are successfully expressed in terms of simple expressions involving quark mass ratios. The chiral flavour symmetry of the family replicated gauge group model is shown to provide the mass protection needed to generate the hierarchical structure of the quark-lepton mass matrices.

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# 1 Introduction

The most important unresolved problem in particle physics is the understanding of flavour and the fermion mass spectrum. The observed values of the fermion masses and mixing angles constitute the bulk of the Standard Model (SM) parameters and provide our main experimental clues to the underlying flavour dynamics. In particular the non-vanishing neutrino masses and mixings provide direct evidence for physics beyond the SM.

The charged lepton masses can be directly measured and correspond to the poles in their propagators:

$$M_e = 0.511 \text{ MeV} \quad M_\mu = 106 \text{ MeV} \quad M_\tau = 1.78 \text{ GeV} \quad (1)$$

However the quark masses have to be extracted from the properties of hadrons and are usually quoted as running masses  $m_q(\mu)$  evaluated at some renormalisation scale  $\mu$ , which are related to the propagator pole masses  $M_q$  by

$$M_q = m_q(\mu = m_q) \left[ 1 + \frac{4}{3}\alpha_3(m_q) \right] \quad (2)$$

to leading order in QCD. The light  $u$ ,  $d$  and  $s$  quark masses are usually normalised to the scale  $\mu = 1 \text{ GeV}$  (or  $\mu = 2 \text{ GeV}$  for lattice measurements) and to the quark mass itself for the heavy  $c$ ,  $b$  and  $t$  quarks. They are typically given [1] as follows<sup>1</sup>:

$$\begin{aligned} m_u(1 \text{ GeV}) &= 4.5 \pm 1 \text{ MeV} & m_d(1 \text{ GeV}) &= 8 \pm 2 \text{ MeV} \\ m_c(m_c) &= 1.25 \pm 0.15 \text{ GeV} & m_s(1 \text{ GeV}) &= 150 \pm 50 \text{ MeV} \\ m_t(m_t) &= 166 \pm 5 \text{ GeV} & m_b(m_b) &= 4.25 \pm 0.15 \text{ GeV} \end{aligned} \quad (3)$$

However we only have an upper limit on the neutrino masses of  $m_{\nu_i} \lesssim 1 \text{ eV}$  from tritium beta decay and from cosmology, and measurements of the neutrino mass squared differences:

$$\Delta m_{21}^2 \sim 5 \times 10^{-5} \text{ eV}^2 \quad \Delta m_{32}^2 \sim 3 \times 10^{-3} \text{ eV}^2 \quad (4)$$

from solar and atmospheric neutrino oscillation data [2].

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<sup>1</sup>Note that the top quark mass,  $M_t = 174 \pm 5 \text{ GeV}$ , measured at FermiLab is interpreted as the pole mass.

The magnitudes of the quark mixing matrix  $V_{CKM}$  are well measured

$$|V_{CKM}| = \begin{pmatrix} 0.9734 \pm 0.0008 & 0.2196 \pm 0.0020 & 0.0036 \pm 0.0007 \\ 0.224 \pm 0.016 & 0.996 \pm 0.013 & 0.0412 \pm 0.002 \\ 0.0077 \pm 0.0014 & 0.0397 \pm 0.0033 & 0.9992 \pm 0.0002 \end{pmatrix} \quad (5)$$

and a CP violating phase of order unity:

$$\sin^2 \delta_{CP} \sim 1 \quad (6)$$

can reproduce all the CP violation data. Neutrino oscillation data constrain the magnitudes of the lepton mixing matrix elements to lie in the following  $3\sigma$  ranges [2]:

$$|U_{MNS}| = \begin{pmatrix} 0.73 - 0.89 & 0.45 - 0.66 & < 0.24 \\ 0.23 - 0.66 & 0.24 - 0.75 & 0.52 - 0.87 \\ 0.06 - 0.57 & 0.40 - 0.82 & 0.48 - 0.85 \end{pmatrix} \quad (7)$$

Due to the Majorana nature of the neutrino mass matrix, there are three unknown CP violating phases  $\delta$ ,  $\alpha_1$  and  $\alpha_2$  in this case [2].

The charged fermion masses range over five orders of magnitude, whereas there seems to be a relatively mild neutrino mass hierarchy. The absolute neutrino mass scale ( $m_\nu < 1$  eV) suggests a new physics mass scale – the so-called see-saw scale  $\Lambda_{seesaw} \sim 10^{15}$  GeV. The quark mixing matrix  $V_{CKM}$  is also hierarchical, with small off-diagonal elements. However the elements of  $U_{MNS}$  are all of the same order of magnitude except for  $|U_{e3}| < 0.24$ , corresponding to two leptonic mixing angles being close to maximal ( $\theta_{atmospheric} \simeq \pi/4$  and  $\theta_{solar} \simeq \pi/6$ ).

We introduce the mechanism of mass protection by approximately conserved chiral charges in section 2. The top quark mass is the dominant term in the SM fermion mass matrix, so it is likely that its value will be understood dynamically before those of the other fermions. In section 3 we discuss the connection between the top quark and Higgs masses and how they can be determined from the so-called Multiple Point Principle. We present the lightest family mass generation model in section 4, which provides an ansatz for the texture of fermion mass matrices and expresses all the quark mixing angles successfully in terms of simple expressions involving quark mass ratios. The family replicated gauge group model is presented in section 5, as an example of a model whose gauge group naturally provides the mass protecting quantum numbers needed to generate the required texture for the fermion mass matrices. Finally we present a brief conclusion in section 6.

## 2 Fermion Mass and Mass Protection

A fermion mass term

$$\mathcal{L}_{mass} = -m\bar{\psi}_L\psi_R + h.c. \quad (8)$$

couples together a left-handed Weyl field  $\psi_L$  and a right-handed Weyl field  $\psi_R$ , which then satisfy the Dirac equation

$$i\gamma^\mu\partial_\mu\psi_L = m\psi_R \quad (9)$$

If the two Weyl fields are not charge conjugates  $\psi_L \neq (\psi_R)^c$  we have a Dirac mass term and the two fields  $\psi_L$  and  $\psi_R$  together correspond to a Dirac spinor. However if the two Weyl fields are charge conjugates  $\psi_L = (\psi_R)^c$  we have a Majorana mass term and the corresponding four component Majorana spinor has only two degrees of freedom. Particles carrying an exactly conserved charge, like the electron carries electric charge, must be distinct from their anti-particles and can only have Dirac masses with  $\psi_L$  and  $\psi_R$  having equal conserved charges. However a neutrino could be a Majorana particle.

If  $\psi_L$  and  $\psi_R$  have different quantum numbers, i.e. belong to inequivalent representations of a symmetry group  $G$  ( $G$  is then called a chiral symmetry), a Dirac mass term is forbidden in the limit of an exact  $G$  symmetry and they represent two massless Weyl particles. Thus the  $G$  symmetry “protects” the fermion from gaining a mass. Such a fermion can only gain a mass when  $G$  is spontaneously broken.

The left-handed and right-handed top quark,  $t_L$  and  $t_R$ , carry unequal Standard Model  $SU(2) \times U(1)$  gauge charges  $\vec{Q}$ :

$$\vec{Q}_L \neq \vec{Q}_R \quad (\text{Chiral charges}) \quad (10)$$

Hence electroweak gauge invariance protects the quarks and leptons from gaining a fundamental mass term ( $\bar{t}_L t_R$  is not gauge invariant). This *mass protection* mechanism is of course broken by the Higgs effect, when the vacuum expectation value of the Weinberg-Salam Higgs field

$$\langle \phi_{WS} \rangle = \sqrt{2}v = 246 \text{ GeV} \quad (11)$$

breaks the gauge symmetry and the SM gauge invariant Yukawa couplings  $\frac{y_i}{\sqrt{2}}$  generate the running quark masses  $m_i = y_i v = 174 y_i$  GeV. In this way a top quark mass of the same order of magnitude as the SM Higgs field vacuum expectation value (VEV) is naturally generated (with  $y_t$  unsuppressed). Thus

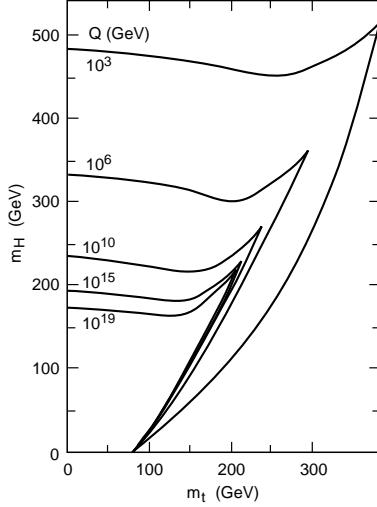


Figure 1: SM bounds in the  $(m_t, m_H)$  plane for various values of  $\Lambda = Q$ , the scale at which new physics enters.

the Higgs mechanism explains why the top quark mass is suppressed, relative to the fundamental (Planck, GUT...) mass scale of the physics beyond the SM, down to the scale of electroweak gauge symmetry breaking. However the further suppression of the other quark-lepton masses ( $y_b, y_c, y_s, y_u, y_d \ll 1$ ) remains a mystery, which it is natural to attribute to mass protection by other approximately conserved chiral gauge charges beyond the SM, as discussed in section 5 for the family replicated gauge group model.

Fermions which are vector-like under the SM gauge group ( $\vec{Q}_L = \vec{Q}_R$ ) are not mass protected and are expected to have a large mass associated with new (grand unified, string,...) physics. The Higgs particle, being a scalar, is not mass protected and *a priori* would also be expected to have a large mass; this is the well-known gauge hierarchy problem discussed at Portoroz by Holger Nielsen [3].

### 3 Top Quark and Higgs Masses from the Multiple Point Principle

It is well-known [4] that the self-consistency of the pure SM up to some physical cut-off scale  $\Lambda$  imposes constraints on the top quark and Higgs boson

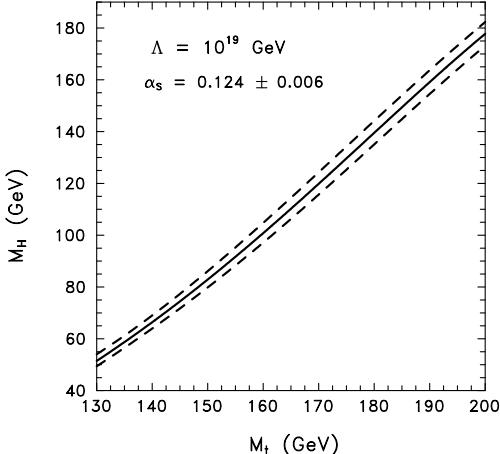


Figure 2: SM vacuum stability curve for  $\Lambda = 10^{19}$  GeV and  $\alpha_s = 0.124$  (solid line),  $\alpha_s = 0.118$  (upper dashed line),  $\alpha_s = 0.130$  (lower dashed line).

masses. The first constraint is the so-called triviality bound: the running Higgs coupling constant  $\lambda(\mu)$  should not develop a Landau pole for  $\mu < \Lambda$ . The second is the vacuum stability bound: the running Higgs coupling constant  $\lambda(\mu)$  should not become negative leading to the instability of the usual SM vacuum. These bounds are illustrated [5] in Fig. 1, where the combined triviality and vacuum stability bounds for the SM are shown for different values of the high energy cut-off  $\Lambda$ . The allowed region is the area around the origin bounded by the co-ordinate axes and the solid curve labelled by the appropriate value of  $\Lambda$ . The upper part of each curve corresponds to the triviality bound. The lower part of each curve coincides with the vacuum stability bound and the point in the top right hand corner, where it meets the triviality bound curve, is the infra-red quasi-fixed point for that value of  $\Lambda$ . Here the vacuum stability curve, for a large cut-off of order the Planck scale  $\Lambda_{Planck} \simeq 10^{19}$  GeV, is important for the discussion of the values of the top quark and Higgs boson masses predicted from the Multiple Point Principle.

According to the Multiple Point Principle (MPP), Nature chooses coupling constant values such that a number of vacuum states have the same energy density (cosmological constant). This fine-tuning of the coupling constants is similar to that of temperature for a mixture of co-existing phases such as ice and water. We have previously argued [6] that baby-universe like theories [7],

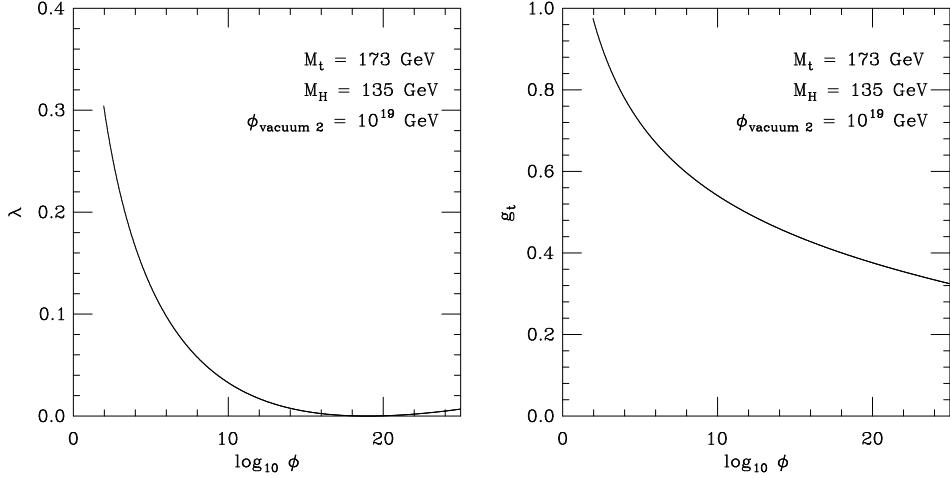


Figure 3: Plots of  $\lambda$  and  $g_t$  as functions of the scale of the Higgs field  $\phi$  for degenerate vacua with the second Higgs VEV at the Planck scale  $\phi_{vac\ 2} = 10^{19}$  GeV. We formally apply the second order SM renormalisation group equations up to a scale of  $10^{25}$  GeV.

having a mild breaking of locality and causality, may contain the underlying physical explanation of the MPP, but it really has the status of a postulated new principle. Here we apply it to the pure Standard Model [8], which we assume valid up close to  $\Lambda_{Planck}$ . So we shall postulate that the effective potential  $V_{eff}(\phi)$  for the SM Higgs field  $\phi$  should have a second minimum, at  $\langle \phi \rangle = \phi_{vac\ 2}$ , degenerate with the well-known first minimum at the electroweak scale  $\langle \phi \rangle = \phi_{vac\ 1} = 246$  GeV:

$$V_{eff}(\phi_{vac\ 1}) = V_{eff}(\phi_{vac\ 2}) \quad (12)$$

Thus we predict that our vacuum is barely stable and we just lie on the vacuum stability curve in the top quark, Higgs particle (pole) mass ( $M_t$ ,  $M_H$ ) plane, shown [9] in Fig. 2 for a cut-off  $\Lambda = 10^{19}$  GeV. Furthermore we expect the second minimum to be within an order of magnitude or so of the fundamental scale, i.e.  $\phi_{vac\ 2} \simeq \Lambda_{Planck}$ . In this way, we essentially select a particular point on the SM vacuum stability curve and hence the MPP condition predicts precise values for  $M_t$  and  $M_H$ .

For large values of the SM Higgs field  $\phi >> \phi_{vac\ 1}$ , the renormalisation group improved tree level effective potential is very well approximated by

$V_{eff}(\phi) \simeq \frac{1}{8}\lambda(\mu = |\phi|)|\phi|^4$  and the degeneracy condition, eq. (12), means that  $\lambda(\phi_{vac\ 2})$  should vanish to high accuracy. The derivative of the effective potential  $V_{eff}(\phi)$  should also be zero at  $\phi_{vac\ 2}$ , because it has a minimum there. Thus at the second minimum of the effective potential the beta function  $\beta_\lambda$  vanishes as well:

$$\beta_\lambda(\mu = \phi_{vac\ 2}) = \lambda(\phi_{vac\ 2}) = 0 \quad (13)$$

which gives to leading order the relationship:

$$\frac{9}{4}g_2^4 + \frac{3}{2}g_2^2g_1^2 + \frac{3}{4}g_1^4 - 12g_t^4 = 0 \quad (14)$$

between the top quark Yukawa coupling  $g_t(\mu)$  and the electroweak gauge coupling constants  $g_1(\mu)$  and  $g_2(\mu)$  at the scale  $\mu = \phi_{vac\ 2} \simeq \Lambda_{Planck}$ . We use the renormalisation group equations to relate the couplings at the Planck scale to their values at the electroweak scale. Figure 3 shows the running coupling constants  $\lambda(\phi)$  and  $g_t(\phi)$  as functions of  $\log(\phi)$ . Their values at the electroweak scale give our predicted combination of pole masses [8]:

$$M_t = 173 \pm 5 \text{ GeV} \quad M_H = 135 \pm 9 \text{ GeV} \quad (15)$$

We have also considered [10] a slightly modified version of MPP, according to which the two vacua are approximately degenerate in such a way that they should both be physically realised over comparable amounts of space-time four volume. This modified MPP corresponds to the Higgs mass lying on the vacuum *metastability* curve rather than on the vacuum stability curve, giving a Higgs mass prediction of  $122 \pm 11$  GeV. We should presumably not really take the MPP predictions to be more accurate than to the order of magnitude of the variation between the metastability and stability bounds. However we definitely predict a light Higgs mass in this range, as seems to be in agreement with indirect estimates of the SM Higgs mass from precision data [1].

This application of the MPP assumes the existence of the hierarchy  $v/\Lambda_{Planck} \sim 10^{-17}$ . Recently we have speculated [11] that this huge scale ratio is a consequence of the existence of yet another vacuum in the SM, at the electroweak scale and degenerate with the two vacua discussed above. The two SM vacua at the electroweak scale are postulated to differ by the condensation of an S-wave bound state formed from 6 top and 6 anti-top quarks mainly due to Higgs boson exchange forces. This scenario is discussed in more detail in Holger Nielsen's talk [3].

## 4 Lightest Family Mass Generation Model

Motivated by the famous Fritzsch ansatz [12] for the two generation quark mass matrices:

$$M_U = \begin{pmatrix} 0 & B \\ B^* & A \end{pmatrix} \quad M_D = \begin{pmatrix} 0 & B' \\ B'^* & A' \end{pmatrix} \quad (16)$$

several ansätze have been proposed for the fermion mass matrices—for example, see [13] for a systematic analysis of symmetric quark mass matrices with texture zeros at the SUSY-GUT scale. Here I will concentrate on the lightest family mass generation model [14]. It successfully generalizes the well-known formula

$$|V_{us}| \simeq \left| \sqrt{\frac{m_d}{m_s}} - e^{i\phi} \sqrt{\frac{m_u}{m_c}} \right| \quad (17)$$

for the Cabibbo angle derived from the above ansatz, eq. (16), to simple working formulae for all the quark mixing angles in terms of quark mass ratios. According to this model the flavour mixing for quarks is basically determined by the mechanism responsible for generating the physical masses of the up and down quarks,  $m_u$  and  $m_d$  respectively. So, in the chiral symmetry limit, when  $m_u$  and  $m_d$  vanish, all the quark mixing angles vanish. Therefore we are led to consider an ansatz in which the diagonal mass matrix elements for the second and third generations are practically the same in the gauge (unrotated) and physical bases.

The mass matrix for the down quarks ( $D = d, s, b$ ) is taken to be hermitian with three texture zeros of the following form:

$$M_D = \begin{pmatrix} 0 & a_D & 0 \\ a_D^* & A_D & b_D \\ 0 & b_D^* & B_D \end{pmatrix} \quad (18)$$

where

$$B_D = m_b + \delta_D \quad A_D = m_s + \delta'_D \quad |\delta_D| \ll m_s \quad |\delta'_D| \ll m_d \quad (19)$$

It is, of course, necessary to assume some hierarchy between the elements, which we take to be:  $B_D \gg A_D \sim |b_D| \gg |a_D|$ . The zero in the  $(M_D)_{11}$  element corresponds to the commonly accepted conjecture that the lightest family masses appear as a direct result of flavour mixings. The zero in  $(M_D)_{13}$  means that only minimal “nearest neighbour” interactions occur,

giving a tridiagonal matrix structure. Since the trace and determinant of the hermitian matrix  $M_D$  gives the sum and product of its eigenvalues, it follows that

$$\delta_D \simeq -m_d \quad (20)$$

while  $\delta'_D$  is vanishingly small and can be neglected in further considerations.

It may easily be shown that equations (18 - 20) are entirely equivalent to the condition that the diagonal elements ( $A_D$ ,  $B_D$ ) of  $M_D$  are proportional to the modulus square of the off-diagonal elements ( $a_D$ ,  $b_D$ ):

$$\frac{A_D}{B_D} = \left| \frac{a_D}{b_D} \right|^2 \quad (21)$$

Using the conservation of the trace, determinant and sum of principal minors of the hermitian matrix  $M_D$  under unitary transformations, we are led to a complete determination of the moduli of all its elements, which can be expressed to high accuracy as follows:

$$|M_D| = \begin{pmatrix} 0 & \sqrt{m_d m_s} & 0 \\ \sqrt{m_d m_s} & m_s & \sqrt{m_d m_b} \\ 0 & \sqrt{m_d m_b} & m_b - m_d \end{pmatrix} \quad (22)$$

The mass matrix for the up quarks is taken to be of the following hermitian form:

$$M_U = \begin{pmatrix} 0 & 0 & c_U \\ 0 & A_U & 0 \\ c_U^* & 0 & B_U \end{pmatrix} \quad (23)$$

The moduli of all the elements of  $M_U$  can also be readily determined in terms of the physical masses as follows:

$$|M_U| = \begin{pmatrix} 0 & 0 & \sqrt{m_u m_t} \\ 0 & m_c & 0 \\ \sqrt{m_u m_t} & 0 & m_t - m_u \end{pmatrix} \quad (24)$$

The CKM quark mixing matrix elements can now be readily calculated by diagonalising the mass matrices  $M_D$  and  $M_U$ . They are given in terms of quark mass ratios as follows:

$$|V_{us}| = \sqrt{\frac{m_d}{m_s}} = 0.222 \pm 0.004 \quad |V_{us}|_{exp} = 0.221 \pm 0.003 \quad (25)$$

$$|V_{cb}| = \sqrt{\frac{m_d}{m_b}} = 0.038 \pm 0.004 \quad |V_{cb}|_{exp} = 0.039 \pm 0.003 \quad (26)$$

$$|V_{ub}| = \sqrt{\frac{m_u}{m_t}} = 0.0036 \pm 0.0006 \quad |V_{ub}|_{exp} = 0.0036 \pm 0.0006 \quad (27)$$

$$|V_{td}| = |V_{us}V_{cb} - V_{ub}| = 0.009 \pm 0.002 \quad |V_{ub}|_{exp} = 0.0077 \pm 0.0014 \quad (28)$$

As can be seen, they are in impressive agreement with the experimental values. The MNS lepton mixing matrix can also be fitted, if the texture of eq. (18) is extended to the Dirac and Majorana right-handed neutrino mass matrices [15].

The proportionality condition, eq. (21), is not so easy to generate from an underlying symmetry beyond the Standard Model, but it is possible to realise it in a local chiral  $SU(3)$  family symmetry<sup>2</sup> model [16].

## 5 Family Replicated Gauge Group Model

As pointed out in section 2, a natural explanation of the charged fermion mass hierarchy would be mass protection due to the existence of some approximately conserved chiral charges beyond the SM. An attractive possibility is that these chiral charges arise as a natural feature of the gauge symmetry group of the fundamental theory beyond the SM. This is the case in the family replicated gauge group model (also called the anti-grand unification model) [18, 19]. The new chiral charges provide selection rules forbidding the transitions between the various left-handed and right-handed quark-lepton states, except for the top quark. In order to generate mass terms for the other fermion states, we have to introduce new Higgs fields, which break the symmetry group  $G$  of the fundamental theory down to the SM group. We also need suitable intermediate fermion states to mediate the forbidden transitions, which we take to be vector-like Dirac fermions with a mass of order the fundamental scale  $M_F$  of the theory. In this way effective SM Yukawa coupling constants are generated [20], which are suppressed by the appropriate product of Higgs field VEVs measured in units of  $M_F$ . We assume that all the couplings in the fundamental theory are unsuppressed, i.e. they are all naturally of order unity.

The family replicated gauge group model is based on a non-simple non-supersymmetric extension of the SM with three copies of the SM gauge

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<sup>2</sup>See ref. [17] for a local chiral  $SU(3)$  family model with an alternative texture.

group—one for each family or generation. With the inclusion of three right-handed neutrinos, the gauge group becomes  $G = (SMG \times U(1)_{B-L})^3$ , where the three copies of the SM gauge group are supplemented by an abelian ( $B - L$ ) (= baryon number minus lepton number) gauge group for each family<sup>3</sup>. The gauge group  $G$  is the largest anomaly free group, transforming the known 45 Weyl fermions plus the three right-handed neutrinos into each other unitarily, which does *not* unify the irreducible representations under the SM gauge group. It is supposed to be effective at energies near to the Planck scale,  $M_F = \Lambda_{Planck}$ , where the  $i$ 'th proto-family couples to just the  $i$ 'th group factor  $SMG_i \times U(1)_{B_i - L_i}$ . The gauge group  $G$  is broken down by four Higgs fields  $W$ ,  $T$ ,  $\rho$  and  $\omega$ , having VEVs about one order of magnitude lower than the Planck scale, to its diagonal subgroup:

$$(SMG \times U(1)_{B-L})^3 \rightarrow SMG \times U(1)_{B-L} \quad (29)$$

The diagonal  $U(1)_{B-L}$  is broken down at the see-saw scale, by another Higgs field  $\phi_{SS}$ , and the diagonal  $SMG$  is broken down to  $SU(3) \times U(1)_{em}$  by the Weinberg-Salam Higgs field  $\phi_{WS}$ .

Table 1: All  $U(1)$  quantum charges of the Higgs fields in the  $(SMG \times U(1)_{B-L})^3$  model.

	$y_1/2$	$y_2/2$	$y_3/2$	$(B - L)_1$	$(B - L)_2$	$(B - L)_3$
$\omega$	$\frac{1}{6}$	$-\frac{1}{6}$	0	0	0	0
$\rho$	0	0	0	$-\frac{1}{3}$	$\frac{1}{3}$	0
$W$	0	$-\frac{1}{2}$	$\frac{1}{2}$	0	$-\frac{1}{3}$	$\frac{1}{3}$
$T$	0	$-\frac{1}{6}$	$\frac{1}{6}$	0	0	0
$\phi_{WS}$	0	$\frac{2}{3}$	$-\frac{1}{6}$	0	$\frac{1}{3}$	$-\frac{1}{3}$
$\phi_{SS}$	0	1	-1	0	2	0

The  $(SMG \times U(1)_{B-L})^3$  gauge quantum numbers of the quarks and leptons are uniquely determined by the structure of the model and they include 6 chiral abelian charges—the weak hypercharge  $y_i/2$  and  $(B - L)_i$  quantum number for each of the three families,  $i = 1, 2, 3$ . With the choice of the

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<sup>3</sup>The family replicated gauge groups  $(SO(10))^3$  and  $(E_6)^3$  have recently been considered by Ling and Ramond [21].

abelian charges in Table 1 for the Higgs fields, it is possible to generate a good order of magnitude fit to the SM fermion masses, with VEVs of order  $M_F/10$ . In this fit, we do not attempt to guess the spectrum of superheavy fermions at the Planck scale, but simply assume a sufficiently rich spectrum to mediate all of the symmetry breaking transitions in the various mass matrix elements. Then, using the quantum numbers of Table 1, the suppression factors are readily calculated as products of Higgs field VEVs measured in Planck units for all the fermion Dirac mass matrix elements<sup>4</sup>, giving for example:

$$M_U \simeq \frac{\langle (\phi_{ws})^\dagger \rangle}{\sqrt{2}} \begin{pmatrix} (\omega^\dagger)^3 W^\dagger T^2 & \omega \rho^\dagger W^\dagger T^2 & \omega \rho^\dagger (W^\dagger)^2 T \\ (\omega^\dagger)^4 \rho W^\dagger T^2 & W^\dagger T^2 & (W^\dagger)^2 T \\ (\omega^\dagger)^4 \rho & 1 & W^\dagger T^\dagger \end{pmatrix} \quad (30)$$

for the up quarks. Similarly the right-handed neutrino Majorana mass matrix is of order:

$$M_R \simeq \langle \phi_{ss} \rangle \begin{pmatrix} (\rho^\dagger)^6 T^6 & (\rho^\dagger)^3 T^6 & (\rho^\dagger)^3 W^3 (T^\dagger)^3 \\ (\rho^\dagger)^3 T^6 & T^6 & W^3 (T^\dagger)^3 \\ (\rho^\dagger)^3 W^3 (T^\dagger)^3 & W^3 (T^\dagger)^3 & W^6 (T^\dagger)^{12} \end{pmatrix} \quad (31)$$

and the effective light neutrino mass matrix can be calculated from the Dirac neutrino mass matrix  $M_N$  and  $M_R$  using the see-saw formula [22]:

$$M_\nu = M_N M_R^{-1} M_N^T \quad (32)$$

In this way we obtain a good 5 parameter fit to the orders of magnitude of all the quark-lepton masses and mixing angles, as given in Table 2, actually even with the expected accuracy [23].

## 6 Conclusion

The hierarchical structure of the quark-lepton spectrum was emphasized and interpreted as due to the existence of a mass protection mechanism, controlled by approximately conserved chiral flavour quantum numbers beyond the SM. The family replicated gauge group model assigns a unique set of anomaly free gauge charges to the quarks and leptons. With an appropriate choice of

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<sup>4</sup>For clarity we distinguish between Higgs fields and their hermitian conjugates.

Table 2: Best fit to quark-lepton mass spectrum. All masses are running masses at 1 GeV except the top quark mass which is the pole mass.

	Fitted	Experimental
$m_u$	4.4 MeV	4 MeV
$m_d$	4.3 MeV	9 MeV
$m_e$	1.6 MeV	0.5 MeV
$m_c$	0.64 GeV	1.4 GeV
$m_s$	295 MeV	200 MeV
$m_\mu$	111 MeV	105 MeV
$M_t$	202 GeV	180 GeV
$m_b$	5.7 GeV	6.3 GeV
$m_\tau$	1.46 GeV	1.78 GeV
$V_{us}$	0.11	0.22
$V_{cb}$	0.026	0.041
$V_{ub}$	0.0027	0.0035
$\Delta m^2_\odot$	$9.0 \times 10^{-5} \text{ eV}^2$	$5.0 \times 10^{-5} \text{ eV}^2$
$\Delta m^2_{\text{atm}}$	$1.7 \times 10^{-3} \text{ eV}^2$	$2.5 \times 10^{-3} \text{ eV}^2$
$\tan^2 \theta_\odot$	0.26	0.34
$\tan^2 \theta_{\text{atm}}$	0.65	1.0
$\tan^2 \theta_{\text{chooz}}$	$2.9 \times 10^{-2}$	$\lesssim 2.6 \times 10^{-2}$

quantum numbers for the Higgs fields, these chiral charges naturally generate a realistic set of quark-lepton masses and mixing angles. The top quark dominates the fermion mass matrices and we showed how the Multiple Point Principle can be used to predict the top quark and SM Higgs boson masses. We also discussed the lightest family mass generation model, which gives simple and compact formulae for all the CKM mixing angles in terms of the quark masses.

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